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ELECTROHYDRODYNAMIC FLOW IN A RETARDING ELECTRIC FIELD WITH ALLOWANCE FOR CHARGED PARTICLE INERTIA

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The one-dimensional electrohydrodynamic flow is considered in a retarding electric field when, owing to the inertia of charged particles, it is necessary to use the complete equation of momenta for the charged component. It is shown that in spite of the negligibly small relative volume occupies by particles in the initial cross section of the stream, there is a section in which particles move at low but finite velocity with a finite relative volume of the charged component, when the particles are retarded by an external homogeneous electric field (without taking into account the induced field). Interaction of the charged and neutral components, which may be absent in the initial cross section, is always substantial. The pressure drop required for such flow is determined. The flow is investigated with allowance for induced electric fields that diminish the effect of concentration increase of charged particles.

1. We consider the one-dimensional motion of a two-phase medium of which the carrier phase is a neutral incompressible fluid (gas) of density ρ and true density $\rho^\circ = \mathrm{const}$ flowing at velocity u, and the dispersed phase of density ρ_s consisting of particles of radius a, of density $\rho_s^\circ = \mathrm{const}$ and electric charge Q, flowing at velocity u_s. The number of particles in a unit of volume is n_s. At the cross section $x = 0$ the gas velocity is $u = U = \mathrm{const}$, and the dispersed phase density and velocity are, respectively, ρ_{s0} and u_{s0}. This means that the gasdynamic circuit contains means for maintaining constant the velocity of gas at various intensities of phase interaction and that a source of charged particles which ensures the specified characteristics ρ_{s0} and u_{s0} existence at the cross section $x = 0$. A longitudinal electric field E which is the sum of the external E_∞ and the induced E_i electric fields is generated between the emitter at $x = 0$ and the collector of charged particles at $x = L$ downstream. The boundary condition for E is specified by E_0 at $x = 0$. For low induced field throughout the interelectrode space the condition $E \equiv E_0$ is specified. Below we consider flows in which the dispersed phase is subjected to retardation by the electric field.

The system of equations that define the one-dimensional flow of a two-phase medium is of the form [1, 2]

$$\rho_s u_s u_s' = -\alpha p' + \rho_s k\psi\,(u - u_s) + \rho_s \varkappa E \qquad (1.1)$$

$$\rho u u' = -(1-\alpha)\,p' - \rho_s k\psi\,(u - u_s) \qquad (1.2)$$

$$\rho_s u_s = \rho_{s0} u_{s0}, \quad \rho u = \rho_0 u_0 \qquad (1.3)$$

$$\rho = \rho^\circ\,(1-\alpha), \quad \rho_s = \rho_s^\circ \alpha \qquad (1.4)$$

$$E' = 4\pi\varkappa\rho_s \tag{1.5}$$
$$u_s(0) = u_{s0},\ \alpha(0) = \alpha_0,\ u(0) = u_0,\ E(0) = E_0 \tag{1.6}$$
$$\alpha = \frac{4}{3}\pi a^3 n_s,\quad k = \frac{9\mu}{2a^2\rho_s^\circ},\quad \varkappa = \frac{Q}{m},\quad m = \frac{4}{3}\pi a^3\rho_s^\circ \tag{1.7}$$

where μ is the dynamic viscosity coefficient of the carrier phase, α is the volume concentration of the dispersed phase, m is the mass of charged particle and the prime denotes differentiation with respect to variable x. Equations (1.1) and (1.2) are the equations of momenta of the dispersed and carrier phases, respectively, (1.3) are the equations of continuity, Eqs. (1.4) establish the relation between the phase densities of media, Eq. (1.5) defines the induced electric field, and the relations (1.6) provide the initial conditions. Parameter ψ is generally a function of the Reynolds number determined by the difference of velocities $|u - u_s|$ and concentration α. The presence of function $\psi(\alpha)$ is due to that in a fairly large concentration of particles their hydrodynamic drag differs from that of a single particle in an unbounded stream. The case of $\psi = 1$ corresponds to the Stokes law of drag of an isolated particle.

The term p_s' which is related to the presence of the proper particle pressure p_s produced by particle collisons in the case of their considerable concentration is omitted in equation (1.1). Assuming, by analogy with the kinetic theory of gases, $p_s = n_s K T_s$, where K is the Boltzmann constant and T_s the "kinetic" temperature of particles, for the relative magnitude of the term p_s' we obtain the estimate

$$\left|\frac{p_s'}{\alpha p'}\right| \sim \frac{p_s}{\alpha p} \sim \frac{n_s T_s}{\alpha N_g T} \sim \frac{3}{4\pi}\frac{T_s}{T}\frac{1}{N_g a^3} \sim \frac{T_s}{T}\frac{10^{-10}}{4} \ll 1 \tag{1.8}$$

where the relation $p = N_g K T$ with N_g denoting the number of microparticles (molecules, atoms, ions) in a unit of volume. Estimate (1.8) was obtained for $N_g = 10^{19}\ \mathrm{cm}^{-3}$ and $a = 10\ \mu\mathrm{m}$ and shows that in a wide range of T_s the proper pressure can be neglected.

It should be noted that the induced electric field determined with the use of Eq. (1.5) owing to finite dimensions of particles, generally differs from the true electric field which must be calculated on the basis of solution of the electrostatic problem for a system consisting of many electrically charged bodies of finite dimensions. Certain methods of solving that problem appeared in [3]. To substantiate Eq. (1.5) we evaluate the ratio B of force Q^2/D^2 exercised on a particular particle by the particle nearest to it (D is the distance between the centers of particles) to the force QE, in which $E = E_\infty + E_i$. We have

$$B_\infty = \frac{Q}{D^2 E_\infty} = 3\frac{E^\circ}{E_\infty}\frac{a^2}{D^2},\quad Q \sim 3E^\circ a^2 \tag{1.9}$$
$$B_i = \frac{Q}{D^2 E_i} \sim \frac{Q}{4\pi n_s D^2 L Q} = \frac{1}{4\pi}\frac{D}{L},\quad n_s \sim D^{-3}$$

The formula for the evaluation of the limit charge Q of a particle of radius a imparted to the particle by the "charging" field E° [4] and, also, the approximate formula that links particle concentration n_s with distance D, are used in the above formulas. The parameter L is the characteristic dimension of the region in which charges are concentrated. The wide range of conditions under which the necessary condition $B = \max.\{B_\infty, B_i\} \ll 1$ is satisfied, is readily seen.

Let the flow at cross section $x = 0$ be defined by the following conditions (obtainable in actual applications) : $n_{s0} = 10^3 \text{ cm}^{-3}$, $E_0 = 10$ kw/cm, $U = 10^3$ cm/s, $u_{s0} \approx U$; $a = 10^{-3}$ cm, $\rho_s^\circ = 5$ gm cm^{-3}, $\rho^\circ = 10^{-3}$ gm cm^{-3}, $\mu \approx 2\cdot 10^{-4}$ gm/cm s, $L = 5$ cm . In this case the basic dimensionless parameters of the problem have the following values:

$$\alpha_0 \approx 4.2\cdot 10^{-6}, \quad \tau = \frac{2\rho_s{}^\circ U a^2}{9\mu L} \approx 1.1 \tag{1.10}$$

$$S = \frac{\rho_{s0}}{\rho_0} = \frac{\alpha_0}{1-\alpha_0}\frac{\rho_s{}^\circ}{\rho^\circ} \approx 2.1\cdot 10^{-2}$$

$$M = \frac{\varkappa E_0}{kU} \sim \frac{E_0 E^\circ a}{2\pi\mu U} \approx 0.8, \quad N = \frac{E_i}{E_0} \sim \frac{9\alpha_0 E^\circ L}{E_0 a} \approx 0.2$$

Parameters τ and M are ratios of the inertial terms and of electrical force, respectively, to force F of interaction between phases in the momentum equation (1. 1) for the dispersed phase, parameter S is the ratio of force F to the remaining terms of Eq. (1. 2) for the carrier phase, and parameter N defines the relative intensity of the induced electric field. The relation between charge Q and field E° used in estimates (1. 10) is the same as in (1. 9). It follows from (1. 10) that the volume concentration of the charged phase and its effect on the motion of the neutral phase may, at least in the initial stage of motion, be neglected without appreciably affecting accuracy $(\alpha_0 \ll 1,\ S \ll 1)$. In the first approximation the induced electric field E_i is unimportant $(N < 1)$. However the flow determined in such approximation in the presence of the external retarding field E_∞ results, as shown below, in erroneous results. This is associated with some particular features of flow in regions where the dispersed phase velocity is low. Hence it is necessary to take into account in such regions the finite volume concentration of particles in spite of its negligibly small magnitude at cross section $x = 0$.

Below we consider various models of flow (in ascending order of complexity) that can be used for defining flows on the basis of estimates (1. 10). The results of their application are compared with data obtained by "exact" calculation based on the solution of the complete system of Eqs. (1. 1) — (1. 6). The structure of regions with abrupt change of parameters of the two-phase stream is investigated.

2. Model $\alpha \equiv 0,\ \rho_s / \rho \ll 1,\ N = 0$. In this case the presence of charged particles does not affect the motion of gas whose velocity is uniform throughout the flow region $(u \equiv U)$. The volume of charged particles is negligibly small, and the flow takes place in the specified external electric field $E_\infty = E_0$ which we assume constant and directed against the stream $(E_0 < 0)$. From (1. 1) — (1. 6) we obtain the equation

$$u_s u_s' = k\psi\,(U - u_s) + \varkappa E_0, \quad u_s(0) = u_{s0} \tag{2.1}$$

whose solution for $\psi = 1$ and condition $(-\varkappa E_0 / (kU) - 1) > 0$ (when the retarding effect of the electric field on particles exceeds that of "dragging" by the neutral medium) is of the form

$$\xi = \frac{x}{L} = \tau\left[\eta \ln \frac{w+\eta}{w_0+\eta} - (w - w_0)\right] \qquad (2.2)$$

$$w = \frac{u_s}{U}, \quad w_0 = \frac{u_{s0}}{U}, \quad \eta = -\frac{\varkappa E_0 + kU}{kU}$$

Formula (2.2) shows that the velocity of particles becomes zero when $\xi = \xi_0^+ = \tau\,[\eta \ln(\eta\,/\,(w_0 + \eta)) + w_0]$, and the solution cannot be continued into region $\xi > \xi_0^+$.

When using this model it is necessary either to locate the runoff of charged particles at cross section $x = x_0^+$ or to investigate the possibility of reversal of the motion of particles after their stopping at $x = x_0^+$. In the latter case the velocity $u_s = v_s$ of particles moving in the reverse direction is also determined by the solution of Eq. (2.1) with the boundary condition $v_s(x_0^+) = 0$. The "reverse"particles trajectory is not the same as defined in (2.2), and the velocity v_s at cross section $x = 0$ is lower than the initial velocity u_{s0}. The densities ρ_s and R_s of the direct and reverse streams of charged particles are determined by the conditions $\rho_s u_s = \rho_{s0} u_{s0}$, $R_s v_s = -\rho_{s0} u_{s0}$. As $x \to x_0^+$ the densities ρ_s and R_s increase as $(x_0^+ - x)^{1/2}$. The distribution of ρ_s and R_s can be used for calculating (as the next following approximation) the induced electric field. The latter proves to be continuous when passing through the cross section $x = x_0^+$ at which no surface charge is generated, since

$$\lim_{\varepsilon \to 0} \int_{x_0^+ - \varepsilon}^{x_0^+} (\rho_s + R_s)\, dx = 0$$

The derived solution which for the proposed model is "exact", from the point of view of the general formulation of the problem proves to be incorrect. This is due to the assumption that the volume concentration is zero up to the cross section $x = x_0^+$, while in the close proximity of that section, where according to (1.3) the phase density of particles becomes considerable and their volume concentration assumes finite values. The abrupt increase of α when the two-phase medium approaches cross section $x = x_0^+$ is accompanied by velocity increase of the carrier phase owing to the decrease of the effective cross section of flow (the stream "contraction" effect). Moreover it is necessary to take into account in the right-hand side of the equation of momenta for the charged phase with finite α, the term $-\alpha p'$ which represents the additional force directed along the x-axis. These effects (friction force increase owing to the increase of velocity $\hat{u}$ of the carrier medium and the appearance of force $-\alpha p'$) substantially alter the pattern of motion. The possibility of particles passing through cross section $x = x_0^+$ emerges. Hence the investigation of the flow pattern near $x = x_0^+$ must use a model which takes into account the finite particle concentration α.

3. **Model** $\alpha \neq 0$, $N = 0$. Under these conditions the parameters of both components undergo changes, and the flow is defined by the system of Eqs. (1.1) – (1.4), and (1.6) in which $E = E_0 = \text{const}$.

In the approximation considered here the effect of the electric field on particles is similar, for instance, to that of gravity. Since the quantity α attains finite values, it is necessary to take into account the dependence $\psi = \psi(\alpha)$. The specification of function $\psi(\alpha)$ defines a "mediated" interaction of particles through the carrier medium. Below, we use for function ψ the following expressions:

$$1^\circ:\ \psi = 1,\quad 2^\circ:\ \psi = (1-\alpha)^{-n},\quad 3^\circ:\ \psi = (1-\alpha/\alpha_m)^{-n},\ n > 0 \tag{3.1}$$

In version 1° the mediated interaction of particles is not taken into account and moving particles are subjected to the Stokes law. Version 2° is borrowed from the fluidized bed theory [1].

Version 3° is a modification of 2°, with α_m representing the maximum volume concentration of particles under conditions of tight packing. The system of Eqs. (1.1) —(1.4) reduces to the single first order differential equation

$$\tau w_\xi' = R(w),\quad w(0) = w_0,\quad \xi = x/L \tag{3.2}$$

$$R(w) = \frac{(w-\delta)}{w}\{-(\eta+1)(w-\delta)^2 + \psi w^2(1+\delta-\alpha_0-w)\} \times \{(w-\delta)^3+\gamma\}^{-1}$$

$$w = \frac{u_s}{U},\quad w_0 = \frac{u_{s0}}{U},\quad \alpha_0 = \frac{\rho_{s0}}{\rho_s^\circ},\quad \eta = -\left(\frac{\varkappa E_0}{kU}+1\right)$$

$$\delta = w_0\alpha_0 \ll 1,\quad \gamma = \frac{\rho^\circ}{\rho_s^\circ}(1-\alpha_0)^2\delta \ll 1$$

where parameter τ is defined by formulas (1.10). We consider the situation when in accordance with (1.10) the volume concentration α_0 of the dispersed phase at the initial cross section is small and the particles motion is retarded by the electric field $E_0 < 0$, $\eta > 0$.

It follows from (3.2) that $R(w_0) < 0$ and the velocity of charged particles along the ξ-axis continuously diminishes until the value $w = w^+$, where w^+ is the nearest to $w = w_0$ positive root of equation $R(w) = 0$, is reached. If then function $R(w)$ in the neighborhood of point $w = w^+$ is of the form $R(w) = \mathrm{const}\,(w - w^+)^m$, $m \geqslant 1$, the value w^+ is reached at $\xi \to \infty$, i.e. there is an "asymptotic" section of motion. The quantity w^+ must be close to zero, since if it were $w^+ = O(1)$, we would have $\alpha^+ = \delta / w^+ = o(1)$ and the flow throughout the region would be defined approximately by $\alpha \equiv 0$ for which function $R(w)$ vanishes when $w < 0$.

We assume n to be an integer and seek the root w^+ in the form of series $w^+ = A\delta + \ldots$. For the determination of A we obtain the algebraic equation

$$\left(1-\frac{1}{A}\right)^2\left(1-\frac{1}{A\alpha_m}\right)^n = \frac{1}{1+\eta} \tag{3.3}$$

whose solution for cases 1° and 2° $(\alpha_m = 1)$ is of the form

$$A = \frac{(1+\eta)^{1/(n+2)}}{(1+\eta)^{1/(n+2)}-1} \tag{3.4}$$

The properties of flow along the asymptotic section (denoted by the superscript +) are defined by formulas

$$\alpha^+ = \frac{1}{A} + O(\delta),\quad w^+ = A\delta + O(\delta^2),\quad \frac{\rho_s}{\rho_{s0}} = \frac{w_0}{A\delta} + O(1) \tag{3.5}$$

$$\frac{u^+}{U} = \frac{A}{A-1} + O(\delta), \quad \frac{\rho^+}{\rho_0} = \frac{A-1}{A} + O(\delta)$$
$$-\left(\frac{p'L}{\rho^\circ U^2}\right)^+ = \frac{\rho_s^\circ}{\rho^\circ \tau} \frac{A^{1+n}}{(A-1)^{2+n}}$$

The asymptotic solution is characterized by the finite volume concentration α^+ of particles, high relative phase density (of order δ^{-1}), the carrier phase velocity $u^+ > U$ (since $A > 1$), density $\rho^+ < \rho_0$, and considerable pressure gradient necessary for obtaining the considered flow.

With strong electric fields the quantity α^+ approaches unity, as implied by (3.5), but in reality it cannot exceed the quantity $\alpha_m = \pi / 6$, i.e. the concentration of closely packed particles. The possibility of occurrence of a solution with $\alpha > \alpha_m$ is due to the imperfection of the "mediated" interaction law in 2°. That imperfection is eliminated in the law 3°. The dependence of the quantity $(A\alpha_m)^{-1}$ on $1 + \eta$ for various n, determined by the solution of Eq. (3.3), is shown in Fig.1. Since the quantity A is greater than $1 / \alpha_m$ for all η, hence $\alpha^+ < \alpha_m$.

Thus in the first approximation the flow consists of two sections. Along the first of these the variation of parameters is defined by formulas (2.2); its length is close to ξ_0^+, and the particle velocity along it decreases virtually to zero. Along the second asymptotic section the parameters of the two-phase stream are defined by formulas (3.5). A narrow transition zone of length of order of δ lies between these sections. Becuase of this, that zone can be replaced by a discontinuity surface along which parameters α, u, ρ abruptly change from $\alpha = 0$, $u = U$, $\rho = \rho_0$ to α^+, u^+; ρ^+, and the velocity w is close to zero.

Figure 2 shows the variation of velocities w and $w_g = u / U$ of the dispersed and carrier phase and of volume concentrations α along the stream, obtained by integrating the system of Eqs. (1.1) – (1.4) for

$$\tau = 1, \; w_0 = 1, \; \alpha_0 = \delta = 3 \cdot 10^{-4}, \; \gamma = 3 \cdot 10^{-7}, \; \eta = 2$$

with the drag law 3° ($n = 3$). The transition zone is shown there in larger scale for the same parameter values.

The above analysis can be extended to the case when the external retarding electric field is not uniform — $\eta = \eta(x)$. Let γ be a parameter of second (or higher) order of smallness with respect to δ. (This condition is satisfied in practice owing to the inequality $\rho^\circ / \rho_s^\circ \ll 1$). It is then possible to neglect the inertia term in Eq. (3.2) for the region in which $w \sim \delta$, and determine w using the condition $R(w) = 0$. The approximate solution of the problem is derived in this case as follows. First, we integrate Eq. (2.1) for $E_0 = E(x)$ and determine the cross section x_0^+ in which velocity u_s vanishes. Flow parameters in region $x > x_0^+$ are determined using formulas (3.5) in which $A = A(x)$ is determined by the solution of Eq. (3.3) for $\eta = \eta(x)$. The cross section at $x = x_0^+$ is the discontinuity surface of parameters α, u and ρ.

In many problems of gasdynamics of multiphase media with the condition $\alpha_0 \ll 1$ satisfied the approximation $\alpha \equiv 0$ is used for describing the whole flow region [5]. The derived above solution shows that in spite of [fulfilment of] condition $\alpha_0 \ll 1$ the allowance for the volume concentration of particles may result in

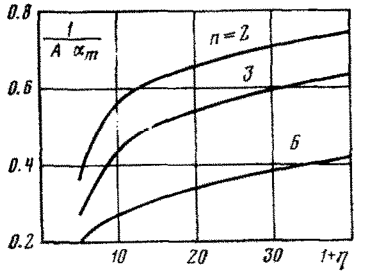

Fig. 1

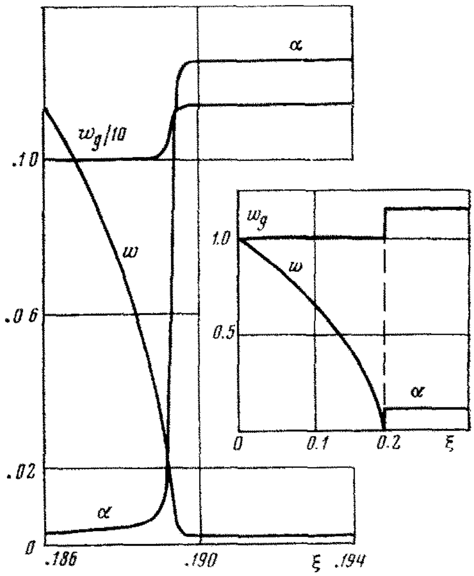

Fig. 2

essentially different flow characteristics in the region of finite dimensions beyond the cross section $x = x_0^+$.

4. **Model** $\alpha \equiv 0, \rho_s / \rho \ll 1, N \sim 1$. The two-phase flow is defined by the system of equations

$$u_s u_s' = k\psi (U - u_s) + \varkappa E \qquad (4.1)$$
$$\rho_s u_s = \rho_{s0} u_{s0}, \quad E' = 4\pi\varkappa\rho_s$$
$$u_s(0) = u_{s0}, \quad E(0) = E_0 < 0$$

whose integral curves $(\psi = 1)$ are shown in Fig. 3 in the plane E, u_s. The arrows indicate the direction of increasing x.

It follows from Fig. 3 that, as in the case considered in Sect. 2, a continuous solution of system (4.1) exists only in region $0 \leqslant x \leqslant x^+$ where $u_s(x^+) = 0$. If x^+ is smaller than the distance between electrodes, it is possible to derive the complete solution within the terms of this model in the following three situations: 1) presence of a "sink" of charged particles at cross section x^+; 2) formation of a surface charge at cross section x^+ (with the electric field discontinuity) and further motion of charged particles downstream; and 3) formation of a surface charge at cross section x_*^+ with absence of particle flow in the region $x > x_*^+$ and presence of "reverse" flow of charged particles in region $(0, x_*^+)$.

The first of these situations is trivial.

When investigating the passage of particles downstream with formation of surface charge at cross section $x = x^+$ (situation 2) it is necessary to analyze the system of Eqs. (4.1) and a similar system for the region $x > x^+$ under conditions

$$u_s(x^+ - 0) = 0, \quad u_s(x^+ + 0) = u_s^+, \quad E(x^+ + 0) = E^+$$

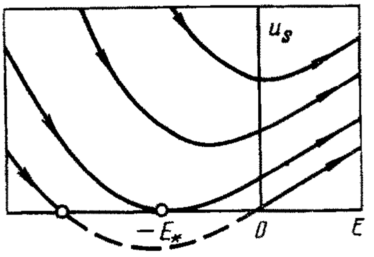

Fig. 3

In the case of situation 3 the system of equations that defines the "direct" and "reverse" motions of charged particles with phase velocities u_s and v_s and densities ρ_s and R_s in the region $0 \leqslant x \leqslant x_*^+$, and the equations for the electric field in the region $x > x_*^+$ are of the form

$$u_s u_s' = k\psi(U - u_s) + \varkappa E, \quad \rho_s u_s = \rho_{s0} u_{s0} \tag{4.2}$$
$$v_s v_s' = k\psi(U - v_s) + \varkappa E, \quad R_s v_s = -\rho_{s0} u_{s0}$$
$$E' = 4\pi\varkappa(\rho_s + R_s)$$
$$u_s(0) = u_{s0}, \quad u_s(x_*^+ - 0) = 0, \quad v_s(x_*^+ - 0) = 0$$
$$E(0) = E_0$$
$$(0 \leqslant x < x_*^+)$$
$$x > x_*^+, \quad E' = 0, \quad E(x_*^+ + 0) = E^+ \tag{4.3}$$

When u_s^+ and E^+ in situation 2, and E^+ in situation 3 are known from solutions of respective systems of equations, the phase velocities and densities of particles, the electric field, and the positions of discontinuity cross sections x^+ and x_*^+ are then determined.

It follows from considerations of the discontinuity surface evolution that the number of boundary conditions at the discontinuity must be five. There are two discontinuities propagating at infinite velocity from the discontinuity of the "electrostatic" perturbation, and two perturbations propagating downstream of the discontinuity, in which the particle density and velocity change.

These conditions (and u^+ and E^+) are to be determined from conditions of c onservation at the discontinuity and the analysis of its structure. By using various structural mechanisms it is possible to obtained different boundary conditions. One of such mechanisms is considered in Sect. 5 on the basis of an investigation of flow with finite volume concentration of particles and the presence of induced electric fields.

5. Model $\alpha \neq 0, N \sim 1$. The distribution of quantities $w, w_g = u / U, \alpha, e = \varkappa E / (kU)$ in the zone of low velocities w and abrupt change of the remaining parameters is shown in Fig. 4. They were obtained by integrating the system of Eqs. (1.1) –(1.6) under conditions

$$\tau = w_0 = 1, \ \alpha_0 = 3\cdot10^{-4}, \ \gamma = 3\cdot10^{-7}, \ \beta = \rho_{s0} 4\pi L\varkappa^2 / (kU) = 3, \ e_0 = \varkappa E_0 / (kU) = -4.5$$

and function ψ conforming to formula (3.1) $(n = 3)$.

There is a section (zone G) of finite length where the dispersed phase velocity is low and the volume concentration α considerable. The size of zone G which is of the order of α_0 depends on the problem parameters. The finiteness of zone G is related to the effect of electrostatic repulsion of charged particles, which in the

Poisson equation for the induced electric field is taken into account.

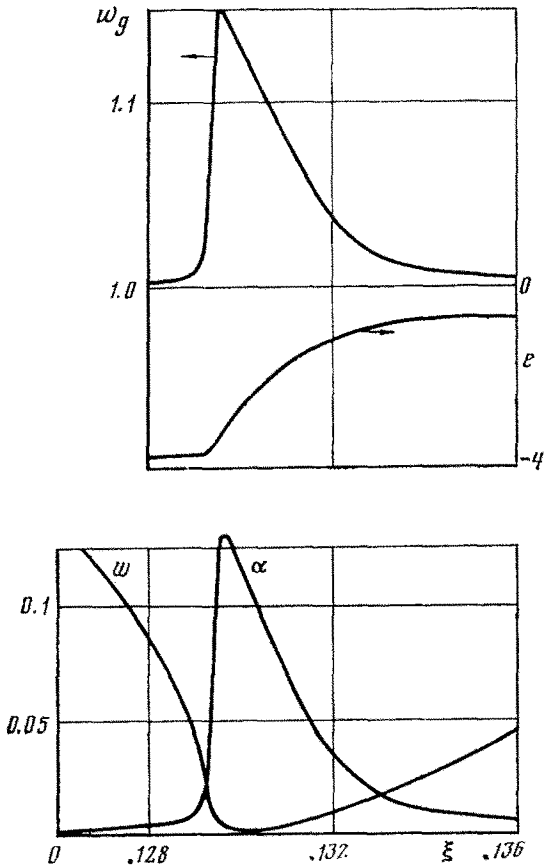

Fig. 4

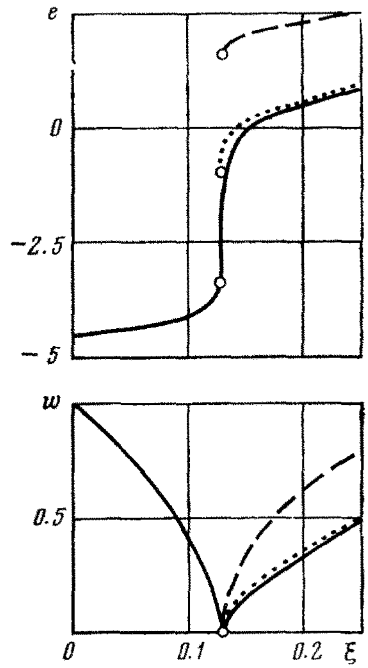

Fig. 5

The considered flow is characterized by a thin layer of considerable concentration of charged particles. The electric field increases abruptly in that zone. The presence of induced electric field (when $\alpha \neq 0$) displaces the cross section of minimal particle velocity upstream, decreases the magnitude of that minimum, and further increase of particle velocity beyond the indicated cross section.

Zone G may be considered as the structural zone of discontinuity which must be introduced when solving the problem in the $\alpha \equiv 0$ approximation. As shown by calculations, this zone consists of two sections: region ω_1 of abrupt increase of α, and w_g the order of magnitude of whose length is that of δ, and of region ω_2 of more smooth variation of parameters in which parameters α and w_g decrease to their "asymptotic" values $\alpha = 0$ and $w_g = 1$.

The investigations described in Sect. 3 had shown that in the zone of low particle velocities (to which the zone ω_2 belongs) the motion of particles can be defined in an "inertia-free" approximation. The particle velocity is then defined as follows:

$$w \approx A\delta, \quad A = A(E) = \frac{(-e)^{1/(n+2)}}{(-e)^{1/(n+2)} - 1}, \quad e = \frac{\varkappa E}{kU} \qquad (5.1)$$

where the formula for A is obtained for $\alpha_m = 1$ and the law of resistance 2°. When $\alpha_m < 1$, parameter A is determined by the solution of Eq. (3.3).

The electric field distribution in the zone of ω_2 is defined by the equation (see (1.5) and (3.5))

$$de / d\xi \sim 1 / (A\delta) \tag{5.2}$$

The asymptotic exist from the zone of ω_2 takes place as $de / d\xi \to 0$ and, consequently, $A \to \infty$. As $A \to \infty$, we have in accordance with (5.1) or (3.3) $-e \to 1$ or $E \to -Uk / \varkappa$, respectively. Note that this condition coincides with the supplementary condition for field E at the discontinuity surface in "classical" electro-gasdynamics, when the inertia of charged particles is neglected [6].

Moreover, in conformity with the first of relations (5.1), velocity w in zone G is a small quantity of order δ, hence it is necessary to set $w^- = 0$, $w^+ = 0$ at the structural zone entry and exit when $\delta \to 0$.

Thus at the discontinuity surface which replaces zone G the relations

$$u_s^- = u_s^+ = 0, \quad \varphi^- = \varphi^+, \quad \rho_s^- u_s^- = \rho_s^+ u_s^+$$
$$U + bE^+ = 0 \quad (b = \varkappa / k) \tag{5.3}$$

are satisfied.

The third of conditions (5.3) for the electric field potential $\varphi = -E'$ is implied by the continuity of the electric field tangential component, and the fourth by the continuity of the dispersed phase mass stream.

Results of numerical integration of the system of Eqs. (4.1) which define the flow of medium in the $\alpha \equiv 0$ approximation in the presence of the discontinuity surface of parameters (5.3) (situation 2 in Sect. 4) are shown in Fig. 5 by dotted lines.

The continuous lines correspond to the solution of the system of Eqs. (1.1) –(1.6) whose particulars in the structural zone are shown in Fig. 4. The dash lines correspond to the numerical integration of Eqs. (4.1) with the first four conditions of (5.3) and the use of the formal condition $E^+ = m > -U / b = m^*$. As expected, the discrepancy between the exact and the approximate solutions is minimal when $m = m^*$.

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